2009 Q10.

(a)

$$\frac{dy}{dx} = \frac{1}{xy} + \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+y^2}{xy}$$

$$\Rightarrow \frac{dy}{dx} \cdot xy = 1+y^2$$

$$\Rightarrow \frac{dy}{dx} \frac{xy}{1+y^2} = 1$$

$$\Rightarrow \frac{y}{1+y^2}\,dy = \frac{1}{x}\,dx$$

$$\Rightarrow \int \frac{y}{1+y^2}\,dy = \int \frac{1}{x}\,dx$$

$$\Rightarrow \frac{1}{2}\ln(1+y^2) = \ln x + C$$

$y = \sqrt{3}$ when $x = 1$

$$\Rightarrow \frac{1}{2}\ln(1+(\sqrt{3})^2) = \ln 1 + C$$

$$\Rightarrow \frac{1}{2}\ln 4 = C$$

$$\Rightarrow \ln 4^{\frac{1}{2}} = C$$

$$\Rightarrow \ln 2 = C$$

$$\frac{1}{2}\ln(1+y^2) = \ln x + \ln 2$$

$$\Rightarrow \frac{1}{2}\ln(1+y^2) = \ln 2x$$

$$\Rightarrow \ln(1+y^2)^{\frac{1}{2}} = \ln 2x$$

$$\Rightarrow (1+y^2)^{\frac{1}{2}} = 2x$$

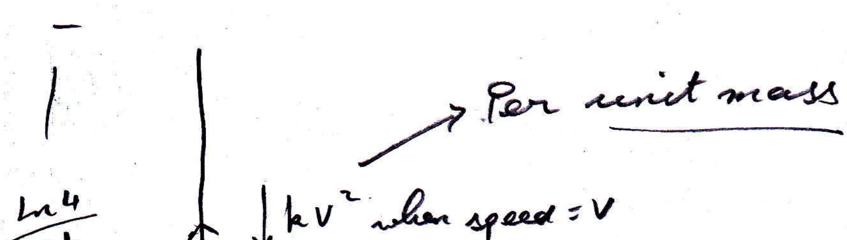
$$\Rightarrow 1+y^2 = 4x^2$$

$$\Rightarrow y^2 = 4x^2-1$$

$$\Rightarrow y = \sqrt{4x^2-1}$$

(b)

(i)



$$F = ma$$

$$\Rightarrow -mg - mkv^2 = m v \frac{dv}{dx}$$

$$\Rightarrow v\frac{dv}{dx} = -g - kv^2$$

$$\Rightarrow \frac{v}{-g-kv^2}\, dv = dx$$

(ii) <u>Again:</u> $F = ma$

$$\Rightarrow -mg - mkV^2 = m\frac{dV}{dt}$$

$$\Rightarrow -(g + kV^2) = \frac{dV}{dt}$$

$$\Rightarrow -(g + kV^2)\,dt = dV$$

$$\Rightarrow dt = \frac{1}{-(g+kV^2)}\,dV$$

$$\Rightarrow \int_0^{\frac{T}{3}} dt = -\int_u^0 \frac{1}{(g+kV^2)}\,dV$$

$$\Rightarrow \left[t\right]_0^{\frac{T}{3}} = \left[-\frac{1}{k}\frac{1}{\sqrt{\frac{g}{k}}}\,\mathrm{Tan}^{-1}\left(\frac{V}{\sqrt{\frac{g}{k}}}\right)\right]_{\sqrt{\frac{3g}{k}}}^{0}$$

<u>since</u> $u = \sqrt{\frac{3g}{k}}$ from (i)

and

$$-\int \frac{1}{g + kV^2} = -\int \frac{1}{k\left(\frac{g}{k} + V^2\right)}$$

$$\left[\begin{array}{l}\underline{\text{Note: P26 Tables}} \\ \int \frac{1}{x^2+a^2} = \frac{1}{a}\mathrm{Tan}^{-1}\frac{x}{a}\end{array}\right]$$

$$= -\frac{1}{k}\int \frac{1}{\left(\sqrt{\frac{g}{k}}\right)^2 + V^2}\cdot dV = -\frac{1}{k}\frac{1}{\sqrt{\frac{g}{k}}}\,\mathrm{Tan}^{-1}\left(\frac{V}{\sqrt{\frac{g}{k}}}\right)$$

$$-\int_u^0 \frac{v}{(g+kv^2)}\,dv = \int_0^{\frac{\ln 4}{2k}} dx$$

$$\Rightarrow \left[\frac{1}{2k}\ln(g+kv^2)\right]_u^0 = \left[x\right]_0^{\frac{\ln 4}{2k}}$$

$$\Rightarrow -\frac{1}{2k}\ln g + \frac{1}{2k}\ln(g+ku^2) = \frac{\ln 4}{2k}$$

$$\Rightarrow -\ln g + \ln(g+ku^2) = \ln 4$$

$$\Rightarrow \ln\left(\frac{g+ku^2}{g}\right) = \ln 4$$

$$\Rightarrow \frac{g+ku^2}{g} = 4$$

$$\Rightarrow g+ku^2 = 4g$$

$$\Rightarrow ku^2 = 3g$$

$$\Rightarrow u = \sqrt{\frac{3g}{k}}$$

$$\Rightarrow \frac{\pi}{3} = +\frac{1}{k}\frac{1}{\sqrt{\frac{g}{k}}} \operatorname{Tan}^{-1}\left(\frac{\sqrt{\frac{3g}{k}}}{\sqrt{\frac{g}{k}}}\right)$$

$$\Rightarrow \frac{\pi}{3} = \frac{1}{\frac{k\sqrt{g}}{\sqrt{k}}} \operatorname{Tan}^{-1}\left(\frac{\frac{\sqrt{3g}}{\sqrt{k}}}{\frac{\sqrt{g}}{\sqrt{k}}}\right)$$

$$\Rightarrow \frac{\pi}{3} = \frac{\sqrt{k}}{k\sqrt{g}} \operatorname{Tan}^{-1} \frac{\sqrt{3g}.\sqrt{k}}{\sqrt{k}\sqrt{g}}$$

$$\Rightarrow \frac{\pi}{3} = \frac{1}{\sqrt{kg}} \operatorname{Tan}^{-1} \sqrt{3}$$

$$\Rightarrow \frac{\pi}{3} = \frac{1}{\sqrt{kg}} . \frac{\pi}{3}$$

$$\Rightarrow 1 = \frac{1}{\sqrt{kg}}$$

$$\Rightarrow \sqrt{kg} = 1$$

$$\Rightarrow kg = 1$$

$$\Rightarrow k = \frac{1}{g}$$