Applied Maths Induction Workshop 4 – Circular Motion and SHM – Exercises

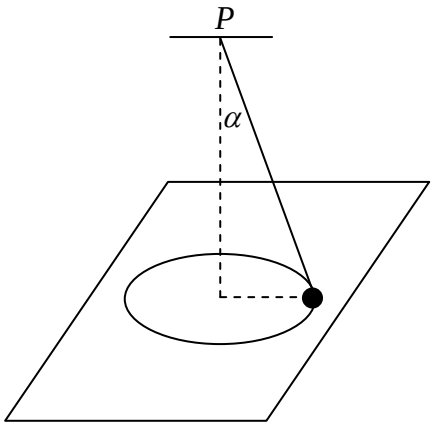
2011 – Ordinary Level – Question 8(a)

A particle describes a horizontal circle of radius 2 m with uniform angular velocity ω radians per second.

Its speed is 8 m s^{-1}.

Find (i) the acceleration of the particle
(ii) the time taken to complete one revolution.

2011 – Ordinary Level – Question 8(b)



A smooth particle of mass 3 kg is attached by a light inelastic string to a fixed point P. The particle describes a horizontal circle or radius $0{\cdot}5 \text{ m}$ on the smooth surface of a horizontal table.
The centre of the circle is vertically below P.

The string makes an angle α with the vertical, where $\tan\alpha = \frac{4}{3}$.

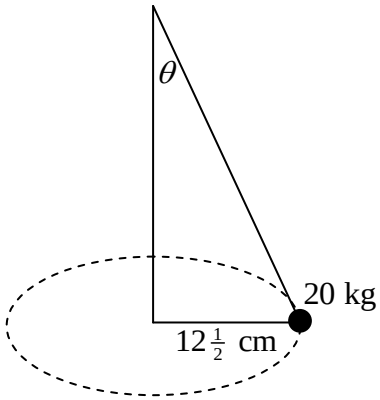
The speed of the particle is 2 m s^{-1}.

Find (i) the tension in the string
(ii) the reaction force between the particle and the table.

2008 – Ordinary Level – Question 8

A particle of mass 20 kg describes a horizontal circle of radius length $12\frac{1}{2}\text{ cm}$ with constant angular velocity of 4rad/s on a smooth horizontal table. The particle is connected by means of a light inextensible string to a fixed point o which is vertically above the centre of the circle.

This inclination of the string to the vertical is θ, where $\tan\theta = \frac{5}{12}$.



(i) Show on a separate diagram all the forces acting on the particle.

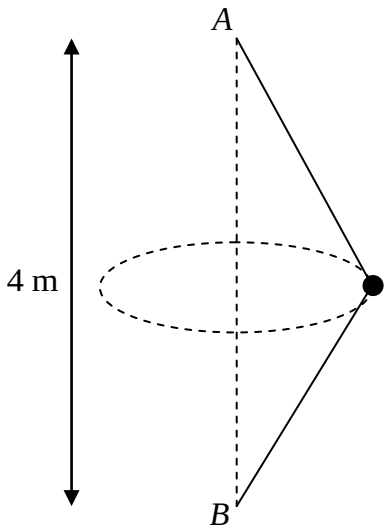
(ii) Show that the value of the normal reaction between the particle and the table is equal to the value of the tension in the string.

2008 – Higher Level – Question 6(b)

A and B are two fixed pegs, A is 4 m vertically above B.

A mass $m\text{ kg}$, connected to A and B by two light inextensible strings of equal length, is describing a horizonttal circle with uniform angular velocity ω.

For what value of ω will the tension in the upper string be double the tension in the lower string?

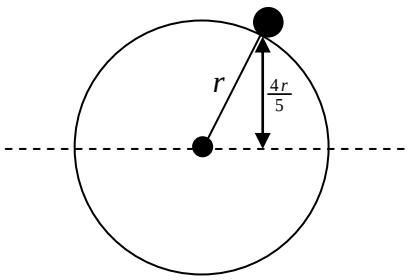


2004 – Higher Level – Question 6(a)

A particle can move on the smooth outer surface of a fixed sphere of radius r.

The particle is released from rest on the smooth surface of the sphere at a height $\frac{4r}{5}$ above the horizontal plane through the centre o of the sphere.

Find, in terms of r, the height above this plane at which the particle leaves the sphere.



1995 – Higher Level – Question 6(b)

A particle of mass m, attached to a fixed point by a light inelastic string, describes a circle in a vertical plane. The tension in the string when the particle is at the highest point of the orbit is T_1 and when at the lowest point it is T_2. Prove that $T_2 = T_1 + 6mg$.

Sample Question

A particle performs SHM with amplitude 10 m and periodic time $2\pi/7$ seconds. Find,

(i) the maximum velocity,

(ii) the maximum acceleration,

(iii) the average speed throughout one full cycle.

2003 – Higher Level – Question 6(a)

A particle is moving with simple harmonic motion of period π seconds about a fixed point o. The maximum speed of the particle is 8cm/s.

(i) Find the amplitude of the motion.

(ii) Find the speed of the particle when it is at a distance of 3 cm from o.

2001 – Higher Level – Question 6(a)

A particle moving with simple harmonic motion has speeds of 5cm/s and 2cm/s when it is at points 3 cm and 4 cm, respectively, from the centre of the motion.

(i) Find the amplitude and the period of the motion.

(ii) Find the maximum speed of the particle.

2006 – Higher Level – Question 6(a)

A particle moves with simple harmonic motion of period 3π. At time $t = 0$, the particle passes through the centre of oscillation. It passes through a point distant 4 m from the centre of motion with a speed of 5m/s away from the centre.

Find, correct to two decimal places,

(i) the maximum acceleration of the particle

(ii) the time which elapses before it next passes through this point.

2007 – Higher Level – Question 6(a)

A particle of mass m kg is suspended from a fixed point p by a light elastic string.
The extension of the string is d when the particle is in equilibrium.
The particle is then displaced vertically from this equilibrium position a distance not greater than d and is then released from rest.

(i) Show that the motion of the particle is simple harmonic.

(ii) Find, in terms of d, the period of the motion.

2011 – Higher Level – Question 6(a)

The distance, x, of a particle from a fixed point, O, is given by

$$x = a\sin(\omega t + \varepsilon)$$

where a, ω and ε are positive constants.

(i) Show that the motion of the particle is simple harmonic.

A particle moving with simple harmonic motion starts from a point 1 m from the centre of the motion with a speed of $9{\cdot}6$ m s^{-1} and an acceleration of 16 m s^{-2}.

(ii) Calculate a, ω and ε.

2010 – Higher Level – Question 6(b)

A particle moves with simple harmonic motion of amplitude $0{\cdot}75$ m.
The period of the motion is 4 s.

Find (i) the maximum speed of the particle

(ii) the time taken by the particle to move from the position of maximum speed to a position at which its speed is half its maximum value.

2004 – Higher Level – Question 6(b)

A particle moves in a straight line such that its displacement from a fixed point o at time t is given by

$$x = a\cos(\omega t - \beta)$$

where a, ω and β are positive constants.

(i) Show that the motion of the particle is simple harmonic motion.
The period of the motion is 16 seconds. At time $t = 4$ s, the particle is 12 m from o and 4 s later the particle is on the other side of o and at a distance of 5 m from o.

(ii) Find a, ω and β.