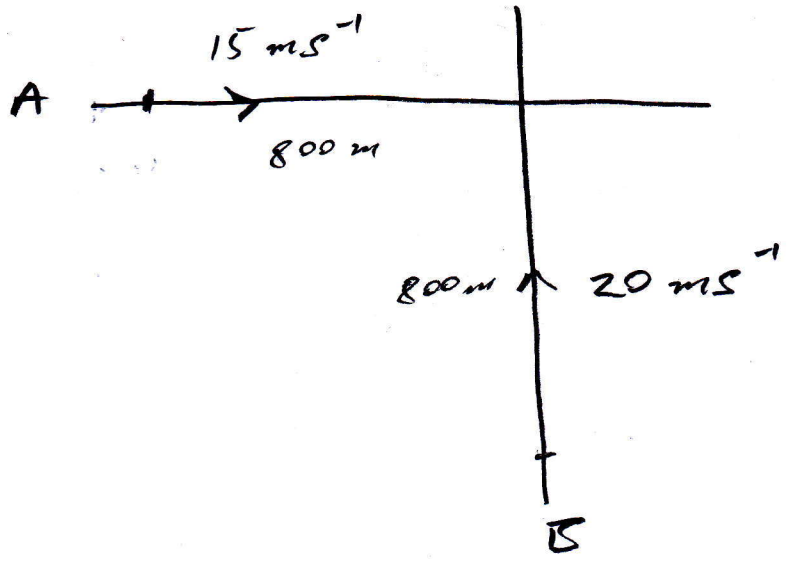
2009 Q2.

$S = \frac{D}{T}$

(a)



(i) $\vec{V_A} = 15\vec{i}$ $\quad \vec{V_B} = 20\vec{j}$

$\Rightarrow \vec{V_{AB}} = \vec{V_A} - \vec{V_B} = 15\vec{i} - 20\vec{j}$

Magnitude $= \sqrt{15^2 + 20^2}$

$= 25 \text{ ms}^{-1}$

Slope $= \frac{-20}{15} = -\frac{4}{3} = \tan\alpha$

$\Rightarrow \alpha = \tan^{-1}\frac{4}{3} = 53.13^\circ$

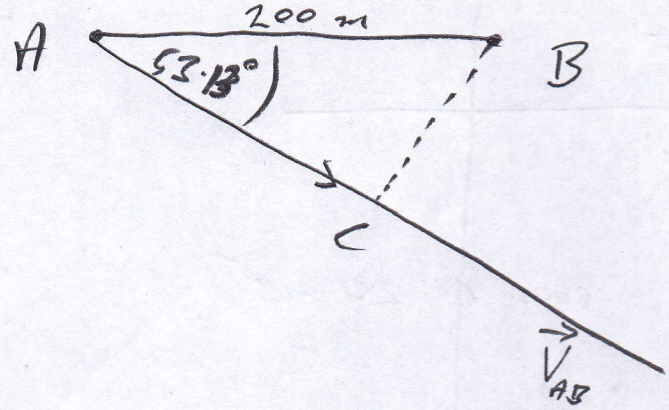
$\Rightarrow$ Direction is E 53.13° S

B is moving faster $\Rightarrow$ will be at the intersection first. It takes B $\frac{800}{20}$ seconds to get there from 800 m away. = 40 sec.

In this time A has travelled (15 × 40) m

= 600 m. $\Rightarrow$ A is 200 m west of B.

[i.e. (800 − 600) m]



$|BC| = 200 \sin 53.13^\circ$

$= 160$ m

(ii) The time it takes A and B to be closest to each other is given by

$$\text{Time} = 40 + \frac{|AC|}{|\vec{V}_{AB}|} = 40 + \frac{200 \cos 53.13^\circ}{25}$$

$= 44.8$ sec.

In this time A has travelled (15×44.8) m $= 672$ m

" " " B " " (20×44.8) m $= 896$ m

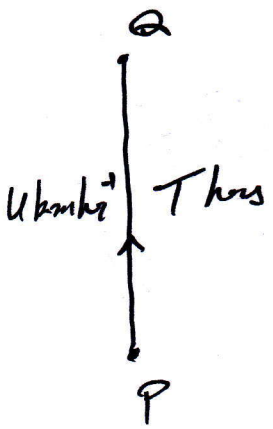
$\Rightarrow$ A is $(800 - 672)$ m from the intersection $= 128$ m

and B " $(800 - 896)$ m " " " $= -96$ m

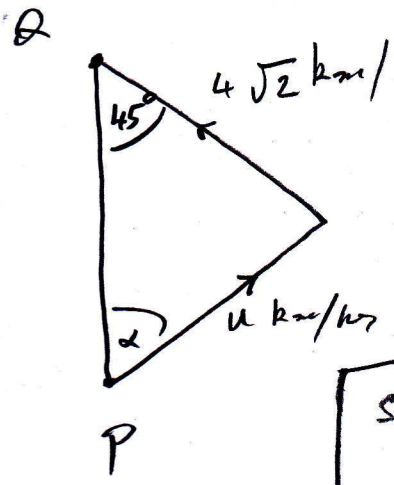
i.e. 96 m past the intersection.

(b)

No Wind:



With wind:



$$s = ut + \frac{1}{2}at^2$$
$$= uT + \frac{1}{2}(0)T^2$$
$$= uT$$

Sine Rule:

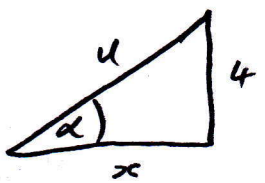
$$\frac{\sin\alpha}{4\sqrt{2}} = \frac{\sin 45}{u}$$

$$\Rightarrow u\sin\alpha = 4\sqrt{2}\sin 45$$

$$\Rightarrow u\sin\alpha = 4\sqrt{2}\cdot\frac{1}{\sqrt{2}}$$

$$\Rightarrow u\sin\alpha = 4$$

$$\Rightarrow \sin\alpha = \frac{4}{u}$$



$$\Rightarrow x = \sqrt{u^2-16}$$

$$\Rightarrow \cos\alpha = \frac{\sqrt{u^2-16}}{u}$$

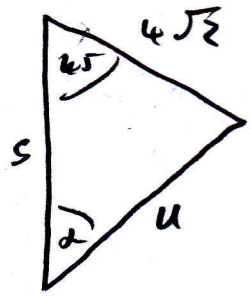
$$\text{Time} = \frac{\text{Dist}}{\text{Speed}} = \frac{\text{Distance in } \vec{j} \text{ direction}}{\text{Speed in } \vec{j} \text{ direction}}$$

$$= \frac{uT}{u\cos\alpha + 4\sqrt{2}\cos\cos 45}$$

or PTO.

Note:

$\vec{i}$ Component is zero



$$\vec{V}_{pw} = u\sin\alpha\,\vec{i} + u\cos\alpha\,\vec{j}$$

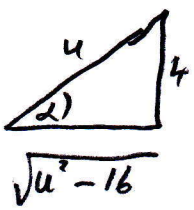
$$\vec{V}_w = -4\sqrt{2}\sin 45\,\vec{i} + 4\sqrt{2}\cos 45\,\vec{j} = -4\vec{i} + 4\vec{j}$$

$$\vec{V}_p = \vec{V}_w + \vec{V}_{pw}$$

$$= (u\sin\alpha - 4)\vec{i} + (u\cos\alpha + 4)\vec{j}$$

$$u\sin\alpha - 4 = 0 \Rightarrow \sin\alpha = \frac{4}{u}$$

$$\Rightarrow \cos\alpha = \frac{\sqrt{u^2-16}}{u}$$



$$\Rightarrow \vec{V}_p = \left(u\cdot\frac{4}{u} - 4\right)\vec{i} + \left(u\,\frac{\sqrt{u^2-16}}{u} + 4\right)\vec{j}$$

$$= 0\,\vec{i} + \left(\sqrt{u^2-16} + 4\right)\vec{j}$$

$$\Rightarrow \text{Magnitude } |\vec{V}_p| = \sqrt{0^2 + \left(\sqrt{u^2-16}+4\right)^2}$$

$$= \sqrt{u^2-16} + 4 \text{ km hr}^{-1}$$

$$\text{Time} = \frac{\text{Dist}}{\text{Speed}} = \frac{uT}{\sqrt{u^2-16}+4}$$